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Degree Equitable Domination in Semigraphs

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Abstract. Let $G = (V, X)$ be a semigraph. The concepts of adjacency domination number, end vertex adjacency domination number and consecutive adjacency domination number of a semigraph is introduced and studied in [6]. Here an attempt is made to study the concept of degree equitable domination number [8] for these three parameters and obtained characterizations for degree equitable minimal (consecutive) adjacency domination number for semigraph.

Key words: Adjacency domination, end vertex adjacency domination, consecutive adjacency domination and degree equitable domination.

AMS Subject classification: 05C69.

1. INTRODUCTION

The concept of semigraph is a natural generalization of graph. The edges of a graph G, can be interpreted in the following two ways.

(i) Each edge uv of G is a 2-element subset of the vertex set V of G.

(ii) Edges of G are 2-tuples (u, v) of vertices of G satisfying the following. Two 2-tuples (u, v) and (u', v') are equal if and only if either $u = u'$ and $v = v'$ or $u = v'$ or $v = u'$.

The Hypergraph theory as developed in Berge [1], generalizes graphs using the first approach. Sampathkumar [7] used the second approach to generalize the graphs which is defined as follows.

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Definition 1.1. *A semigraph G is a pair (V, X) where V is a nonempty set whose elements are called vertices of G and X is a set of n-tuples called the edges of G of distinct vertices, for various $n \geq 2$ satisfying the following conditions.*

(1) *Any two edges have at most one vertex in common and*

(2) *two edges $(u_1, u_2, ...u_n)$and $(v_1, v_2, ..., v_n)$ are considered to be equal if $m = n$ and $u_i = v_i$ for all i, $1 \leq i \leq n$ or $u_i = u_{n-i+1}$ for all i,$1 \leq i \leq n$. Thus the edge $(u_1, u_2, ..., u_n)$ is same as $(u_n, u_{n-1}, ..., n_1)$.*

Let $G = (V, X)$ semigraph. For a vertex $v \in V(G)$, the degree $deg(G)$ is the number of edges having v as an end vertex, adjacent degree $deg_a(G)$ is the number of vertices adjacent to v and consecutive adjacent degree $deg_{ca}(G)$ is the number of vertices which are consecutively adjacent to v. Any two vertices $u, v \in V(G)$ are said to be e-adjacent if they are the end vertices of an edge in G. Similarly, u,v are ca-adjacent if they are consecutively adjacent in G.

Perhaps, the fastest growing area within graph theory is the study of domination and related subset problems, such as independence, covering and matching. Part of the reason for the increased interest in these subset problems, is their many and varied applications in fields such as linear algebra and optimization, design and analysis of communication networks, social sciences, computational complexity and algorithm design.

Definition 1.2. *Let $G = (V, E)$ be a graph. A subset $D \subseteq V$is said to be dominating set for G if every vertex in $V - D$ is adjacent to some vertex in D. The minimum cardinality of such a dominating set is called domination number $\gamma(G)$ of G. For more details refer [4].*

The concept of degree equitable domination is introduced in [8].

Definition 1.3. *A subset $D \subseteq V(G)$ is called an equitable dominating set of G if every vertex $v \in V(G)$ D has a neighbor $u \in D$ such that $|d_G(u) - d_G(v)| \leq 1$. The equitable domination number of G, denoted by $\gamma^e(G)$, is the minimum cardinality of an equitable dominating set of G.*

In [6] the concept of domination as applied to semigraphs and studied various types of domination parameters for semigraphs.

Definition 1.4. *Let $G = (V, X)$ be a semigraph. A set $D \subseteq V$ is called adjacent dominating set (ad-set) if for every vertex $v \in V - D$ there exist a vertex $u \in D$ such that u is adjacent to v. The adjacency domination number $\gamma_a(G)$ of G is the minimum cardinality of an adjacent dominating set of G.*

Definition 1.5. *Let V_e be the set of all end vertices in G. A set $D \subset V_e$ is called end vertex adjacency dominating set (ead-set) if (i) D is an ad-set and (ii) every end vertex $v \in V - D$ is e-adjacent to some vertex $u \in D$ in G. The end vertex adjacency domination number $\gamma_{ea}(G)$ of G is the minimum cardinality of an ead-set of G*

Definition 1.6. *A set $D \subseteq V$ is called consecutive adjacent dominating set (cad-set) if for every vertex $v \in V - D$ there exists a vertex $u \in D$ such that u is ca-adjacent to v in G. The consecutive adjacency domination number $\gamma_{ca}(G)$ of G is the minimum cardinality of cad-set of G.*

In this paper, we have introduced the concept degree equitable domination for the ad-set, ead-set and cad-set for semigraph.

Definition 1.7. *Let $G = (V, X)$ be a semigraph. A set $D \subseteq V$ is called degree equitable adjacent dominating set (ad^e -set) if for every vertex $v \in V - D$ there exist a vertex $u \in D$ such that u is adjacent to v and $|d_G(u) - d_G(v)| \leq 1$.The degree equitable adjacency domination number $\gamma_a^e(G)$ of G is the minimum cardinality of ad^e-set of G.*

Definition 1.8. *Let V_e be the set of all end vertices in G. A set $D \subset Ve$ is called degree equitable end vertex adjacency dominating set (ead^e -set) if (i) D is an ad^e -set and (ii) every end vertex $v \in V - D$ is e-adjacent to some vertex $u \in D$in G and $|d_G(u) - d_G(v)| \leq 1$. The degree equitable end vertex adjacency domination number $\gamma_{ea}^e(G)$ of G is the minimum cardinality of an ead^e -set of G.*

Definition 1.9. *A set $D \subseteq V$ is called degree equitable consecutive adjacent dominating set (cad^e -set) if for every vertex $v \in V - D$ there exists a vertex $u \in D$ such that u is ca-adjacent to v in G and $|d_G(u) - d_G(v)| \leq 1$. The degree equitable consecutive adjacency domination number $\gamma_{ca}^e(G)$ of G is the minimum cardinality of cad^e -set of G.*

Results 1.10. *First we begin with* $\gamma_a^e, \gamma_{ca}^e and \gamma_{ea}^e$ *of some standard class of semigraphs.*

Proposition 1.11. *If G is complete semigraph or strongly complete semigraph, Then*

(i) $\gamma_a^e(K_p) = 1 = \gamma_a^e(K_p^{p-1})$

(ii) $\gamma_{ca}^e(K_p) = 1 = \gamma_{ca}^e(K_p^{p-1})$

(iii) $\gamma_{ea}^e(K_p) = 1 = \gamma_{ea}^e(K_p^{p-1})$

Proposition 1.12. *Let* P_k *denote path semigraph and* C_k *, cycle semigraph with k-vertices containing n end vertices and u middle vertices such that* $m+n=1$ *and let* $G_k = P_k$ *or* C_k *. Then*

(i) $\gamma_e a(G_k) = \gamma_{ea}^e(G_k) \leq \lceil \frac{k}{3} \rceil$

(ii) $\gamma_{ca}^e(G_k) = \lceil \frac{k}{3} \rceil$

Proposition 1.13. *Let* $E_i = (u_1, u_2, ..., u_m)$ *be an edge of semigraph* $G = (V, K)$ *(i.e)* $E_i \in X(G)$*. If* $G = mE_i$ *where* $m \geq 1$ *then*

(i) $\gamma_a^e(G) = m$

(ii) $\gamma_a^{ea}(G) = m$

(iii) $\gamma_{ca}^e(G) = m \lceil \frac{m}{3} \rceil$

Before going to next results we define the following different types of regular semigraphs.

Definition 1.14. *Let* $G = (V, K)$ *be a semigraph. If for every* $v \in V$ *,* $deg_a(V) = k$ *for some positive integer k. Then G is called k-regular adjacency semigraph.*

Similarly, k-regular consecutive adjacency semigraph and k-regular end vertex adjacency graphs are defined. Let $G = (V, X)$ *be a semigraph. One can associate three different graphs with G each having the same vertex set V of G, as follows:*

(i) *The end vertex graph* G_e *: Two vertices in* G_e *are adjacent if and only if they are end vertices of an edge in G.*

(ii) *The adjacency graph* G_a *: Two vertices in*G_a *are adjacent if and only if they are adjacent in G.*

(iii) *The consecutive adjacency graph G_{ca} : Two vertices inG_{ca} are adjacent if and only if they are consecutively adjacent vertices in G.*

Theorem 1.15. *If G is a k- regular adjacency semigraph or k-regular consecutive adjacency semigraph or k-regular end vertex adjacency graph, then*

(i) $\gamma_a^e(G) = \gamma_a(G)$

(ii) $\gamma_a^{ea}(G) = \gamma_{ea}(G)$

(iii) $\gamma_a^{ca}(G) = \gamma_{ca}(G)$

Proof. (1). Let D be a minimum adjacency dominating set of G. Let $u \in V(G)\ D$. Thus there exist vertices $w, v \in D$ such that $uw, uv \in E(G)$. We have $|d_G(u) - d_G(v)| \leq 1$ and $|d_G(u) - d_G(w)| \leq 1$. Therefore D is a degree equitable adjacency dominating set of G. Consequently,$\gamma_a^e(G) \leq |D| = \gamma_a(G)$. Obviously, $\gamma_a(G) \leq \gamma_a^e(G)$. This implies that $\gamma_a^e(G) = \gamma_a(G)$
Similarly we can prove (2) and (3). □

Next, we present the characterizations for degree equitable minimal consecutive adjacency dominating set and degree equitable minimal adjacency dominating set for a semigraph G.

Theorem 1.16. *The degree equitable consecutive dominating set D is minimal if and only if for every vertex $u \in D$ one of the following conditions hold.*

1. *Either $N_{ca} \cup D = \emptyset$ or$|deg_{ca}(v) - deg_{ca}(u)| \geq 2$ for all $N_{ca}(u) \cup D$*
2. *There exists a vertex $v \in V - D$ such that $N_{ca} \cup D = u$ and $|deg_{ca}(v) - deg_{ca}(u)| \leq 1$*

Proof. Let G be a semigraph and D be a degree equitable minimal consecutive adjacency dominating set of G. Therefore D is also degree equitable minimal dominating set of consecutive adjacency graph G_{ca} . It is suffice to prove that D should satisfy the either of the following conditions.

a. Either $N_{G_{ca}} \cap D = \emptyset$ or for all $|deg_{G_{ca}}(v) - deg_{G_{ca}}(u)| \geq 2$ for all $N_{G_{ca}}(u) \cup D$.
b. There exists a vertex $v \in V - D$ such that $N_{G_{ca}}(v) \cup D = u$ and $|deg_{G_{ca}}(v) - deg_{G_{ca}}(u)| \leq 1$

Now assume that D is a minimal equitable dominating set of G_{ca}. Suppose (a) and (b) do not hold. Then for some $u \in D$ there exists a vertex $v \in N_{G_{ca}}(u) \cap D$ such that $|deg_{G_{ca}}(v) - deg_{G_{ca}}(u)| \leq 1$ and for every $v \in V - D$ either $N_{G_{ca}}(v) \cap D \neq u$ or $|deg_{G_{ca}}(v) - deg_{G_{ca}}(u)| \geq 2$ or both. Therefore $D - u$ is an equitable dominating set, a contradiction to the minimality of D. Therefore (a) and (b) holds.

Conversely, suppose for every $u \in D$, one of the statements (a) and (b) holds. Suppose D is minimal. Then there exists a vertex $u \in D$ such that $D - u$ is an equitable dominating set for G_{ca}. Therefore there exists a vertex $v \in D - u$such that v equitably dominates u. That is $v \in N_{G_{ca}} u$ and $|deg_{G_{ca}}(v) - deg_{G_{ca}}(u)| \leq 1$. Therefore u does not satisfy (a). Then u must satisfy (b). Then there exists a vertex $u \in V - D$ such that $N_{G_{ca}} = u$ and $|deg_{G_{ca}}(v) - deg_{G_{ca}}(u)| \leq 1$. Since $D - u$ is an equitable dominating set, there exists $w \in D - u$ such that w is adjacent to v and it is degree equitable with v.

Therefore $w \in N_{G_{ca}}(v) \cap D, |deg_{G_{ca}}(v) - deg_{G_{ca}}(u)| \leq 1$ and $w \neq u$ a contradiction to $N_{G_{ca}}(v) \cap D = u$. Therefore D is a minimal dominating set for G_{ca}. □

Next theorem gives the characterization for degree equitable minimal adjacency dominating sets for semigraph G.

Theorem 1.17. *The degree equitable adjacency dominating set D is minimal if and only if for every vertex $u \in D$ one of the following conditions hold.*

1. *If every$u_i \in D$ is independent then either$N_a(u) \cup D = \emptyset$ or $|deg_{ca}(v) - deg_{ca}(u)| \geq 2$ for all $N_a(u) \cup D$.*
2. *If any two vertices x_i and y_j belongs to same edge E_i and $x_i y_j \in D$. Then $N_a(x_i) \cap D = \{y_j\}$ or$N_a(y_j) \cap D = \{x_i\}$ and $|deg_{G_{ca}}(v) - deg_{G_{ca}}(u)| \leq 1$.*

Proof. We can prove this by a similar way as in the proof of Theorem 2. □

Similarly we can obtain characterization for degree equitable minimal end vertex adjacency dominating sets for semigraph.

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