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Stability of Decic Functional Equations in Multi-Banach Spaces

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Abstract. In this paper, we prove the Stability of Decic Functional Equation in Multi-Banach Spaces by using fixed point technique.

Key words: Hyers-Ulam stability, Multi-Banach Spaces, Decic Functional Equation, Fixed Point Method.

AMS Subject classification: 39B52, 32B72, 32B82, 47H10.

1. Introduction

In 1940, Ulam posed a problem concerning the stability of functional equations:Give conditions in order for a linear function near an approximately linear function to exist. An earlier work was done by Hyers [6] in order to answer Ulam's equation [15] on approximately additive mappings.

During last decades various stability problems for large variety of functional equations have been investigated by several mathematicians. A large list of references concerning in the stability of functional equations can be found. e.g.([1], [2], [6], [7], [9]).

In 2010, Liguang Wang, Bo Liu and ran Bai [10] proved the stability of a mixed type functional equations on Multi - Banach Spaces. In 2010, Tian Zhou Xu, John Michael Rassias, Wan Xin Xu [14] investigated the generalized Ulam-Hyers stability of the general mixed additive-quadratic-cubic-quartic functional equation

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$$f(x+ny)+f(x-ny) = n^2 f(x+y) + n^2 f(x-y) + 2(1-n^2)f(x) + \frac{n^4-n^2}{12}\left[f(2y)+f(-2y)-4f(y)-4f(-y)\right]$$

for fixed integers n with $n \neq 0, \pm 1$ in Multi- Banach Spaces.

In 2011, Zhihua Wang, Xiaopei Li and Th. M. Rassias [17] proved the Hyers - Ulam stability of the additive - cubic - quartic functional equations

$$11\left[f(x+2y)+f(x-2y)\right] = 44\left[f(x+y)+f(x-y)\right] + 12f(3y) - 48f(2y) + 60f(y) - 66f(x)$$

in Multi - Banach Spaces by using fixed point method.

In 2013, Fridoun Moradlou [5] proved the generalized Hyers-Ulam-Rassias stability of the Euler-Lagrange-Jensen Type Additive mapping in Multi-Banach Spaces.

In 2015, Xiuzhong Yang, Lidan Chang, Guofen Liu [16] estabilished the orthogonal stability of mixed additive-quadratic jensen type functional equation in Multi-Banach Spaces.

In 2016, John M. Rassias, M. Arunkumar, E. Sathya and T. Namachivayam [8] established the general solution and also proved the stability of the nonic functional equations

$$f(x+5y) - 9f(x+4y) + 36f(x+3y) - 84f(x+2y) + 126f(x+y) - 126f(x) + 84f(x-y) - 36f(x-2y) + 9f(x-3y) - f(x-4y) = 9!f(y)$$

where $9! = 362880$ in Felbin's type fuzzy normed space and intuitionistic fuzzy normed space using direct and fixed point method.

In 2016, Mohan Arunkumar, Abasalt Bodaghi, John Michael Rassias and Elumalai Sathya [12] proved the general solution of (1) and also proved the stability in Banach spaces, generalized 2-normed spaces and random normed spaces by using direct and fixed point approach.

In this paper, we carry out the following Stability of Decic Functional Equations

$$\begin{aligned}\mathcal{G}f(x,y) = {} & f(x+5y) - 10f(x+4y) + 45f(x+3y) - 120f(x+2y) \\ & + 210f(x+y) - 252f(x) + 210f(x-y) - 120f(x-2y) \\ & + 45f(x-3y) - 10f(x-4y) + f(x-5y) - 10!f(y) \qquad (1)\end{aligned}$$

where $10! = 3628800$ in Multi-Banach Spaces by using fixed point technique.

thm 1.1. *[3], [13] Let $(\mathcal{X}, d)$ be a complete generalized metric space and let $\mathcal{J} : \mathcal{X} \to \mathcal{X}$ be a strictly contractive mapping with Lipschitz constant $\mathcal{L} < 1$. Then for each given element $x \in \mathcal{X}$, either*

$$d(\mathcal{J}^n x, \mathcal{J}^{n+1} x) = \infty$$

for all nonnegative integers n or there exists a positive integer n_0 such that
(i) $d(\mathcal{J}^n x, \mathcal{J}^{n+1} x) < \infty$ for all $n \geq n_0$;
(ii) The sequence $\{\mathcal{J}^n x\}$ is convergent to a fixed point y^* of $\mathcal{J}$;
(iii) y^* is the unique fixed point of T in the set $Y = \{y \in X : d(\mathcal{J}^{n_0} x, y) < \infty\}$;
(iv) $d(y, y^*) \leq \frac{1}{1-\mathcal{L}} d(y, \mathcal{J}y)$ for all $y \in Y$..

Now, let us recall regarding some concepts in Multi-Banach spaces.

Let $(\wp, \|.\|)$ be a complex normed space, and let $k \in \mathbb{N}$. We denote by $\wp^k$ the linear space $\wp \oplus \wp \oplus \wp \oplus \cdots \oplus \wp$ consisting of $k-$ tuples $(x_1, \cdots, x_k)$ where $x_1, \cdots, x_k \in \wp$. The linear operations on $\wp^k$ are defined coordinate wise. The zero element of either $\wp$ or $\wp^k$ is denoted by 0. We denote by $\mathbb{N}_k$ the set $\{1, 2, \cdots, k\}$ and by Ψ_k the group of permutations on k symbols.

Definition 1.2. [4] A Multi- norm on $\{\wp^k : k \in \mathbb{N}\}$ is a sequence $(\|.\|) = (\|.\|_k : k \in \mathbb{N})$ such that $\|.\|_k$ is a norm on $\wp^k$ for each $k \in \mathbb{N}$, $\|x\|_1 = \|x\|$ for each $x \in \wp$, and the following axioms are satisfied for each $k \in \mathbb{N}$ with $k \geq 2$:

(1) $\left\|\left(x_{\sigma(1)}, \cdots, x_{\sigma(k)}\right)\right\|_k = \|(x_1 \cdots x_k)\|_k$, for $\sigma \in \Psi_k, x_1, \cdots, x_k \in \wp$;

(2) $\|(\alpha_1 x_1, \cdots, \alpha_k x_k)\|_k \leq (\max_{i \in \mathbb{N}_k} |\alpha_i|) \|(x_1 \cdots x_k)\|_k$
for $\alpha_1 \cdots \alpha_k \in \mathbb{C}, x_1, \cdots, x_k \in \wp$;

(3) $\|(x_1, \cdots, x_{k-1}, 0)\|_k = \|(x_1, \cdots, x_{k-1})\|_{k-1}$, for $x_1, \cdots, x_{k-1} \in \wp$;

(4) $\|(x_1, \cdots, x_{k-1}, x_{k-1})\|_k = \|(x_1, \cdots, x_{k-1})\|_{k-1}$ for $x_1, \cdots, x_{k-1} \in \wp$.

In this case, we say that $\left((\wp^k, \|.\|_k) : k \in \mathbb{N}\right)$ is a multi - normed space.

Suppose that $\left((\wp^k, \|.\|_k) : k \in \mathbb{N}\right)$ is a multi - normed spaces, and take $k \in \mathbb{N}$. We need the following two properties of multi - norms. They can be found in [4].

$$(a)\ \|(x, \cdots x)\|_k = \|x\|, \text{for} x \in \wp,$$

$$(b) \max_{i \in \mathbb{N}_k} \|x_i\| \leq \|(x_1, \cdots, x_k)\|_k \leq \sum_{i=1}^{k} \|x_i\| \leq k \max_{i \in \mathbb{N}_k} \|x_i\|, \text{for} x_1, \cdots, x_k \in \wp.$$

It follows from (b) that if $(\wp, \|.\|)$ is a Banach space, then $(\wp^k, \|.\|_k)$ is a Banach space for each $k \in \mathbb{N}$; In this case, $\left((\wp^k, \|.\|_k) : k \in \mathbb{N}\right)$ is a multi - Banach space.

thm 1.3. *Let $\mathcal{X}$ be an linear space and let $\left((\mathcal{Y}^k, \|.\|) : K \in \mathbb{N}\right)$ be a multi-Banach space. Suppose that δ is a nonnegative real number and $f : \mathcal{X} \to \mathcal{Y}$ is a mapping satisfying*

$$\sup_{k \in \mathbb{N}} \|(\mathcal{G}f(x_1, y_1), \cdots, \mathcal{G}f(x_1, y_1))\|_k \leq \delta \tag{2}$$

$x_1, \cdots, x_k, y_1, \cdots, y_k \in \mathcal{X}$. *Then there exists a unique decic mapping $\mathcal{D} : \mathcal{X} \to \mathcal{Y}$ such that*

$$\sup_{k \in \mathbb{N}} \|(f(x_1) - \mathcal{D}(x_1), \cdots, f(x_k) - \mathcal{D}(x_k))\|_k \leq \frac{41}{148490496}\delta \tag{3}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$.

Proof. Letting $x_1 = x_2 =, \cdots, = x_k = 0$ and replacing $y_1, \cdots, y_k$ by $2x_1, \cdots, 2x_k$ in (2), we obtain that

$$\begin{aligned}\sup_{k \in \mathbb{N}} \|(&2f(10x_1) - 20f(8x_1) + 90f(6x_1) - 240f(4x_1) - 3628380f(2x_1), \cdots, \\ &2f(10x_k) - 20f(8x_k) + 90f(6x_k) - 240f(4x_k) - 3628380f(2x_k))\|_k \leq \delta\end{aligned} \tag{4}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$.

Dividing by 2 in the above equation, we get

$$\begin{aligned}\sup_{k \in \mathbb{N}} \|(&f(10x_1) - 10f(8x_1) + 45f(6x_1) - 120f(4x_1) - 1814190f(2x_1), \cdots, \\ &f(10x_k) - 10f(8x_k) + 45f(6x_k) - 120f(4x_k) - 1814190f(2x_k))\|_k \leq \frac{1}{2}\delta\end{aligned} \tag{5}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. Again we taking $x_1, \cdots, x_k$ by $5y_1, \cdots 5y_k$ and replacing $y_1, \cdots, y_k$ by $x_1, \cdots x_k$ in (2), we get

$$\begin{aligned}\sup_{k\in\mathbb{N}} \|(f(10x_1) - 10f(9x_1) + 45f(8x_1) - 120f(7x_1) + 210f(6x_1) - 252f(5x_1) \\ +210f(4x_1) - 120f(3x_1) + 45f(2x_1) - 3628810f(x_1), \cdots, f(10x_k) \\ -10f(9x_k) + 45f(8x_k) - 120f(7x_k) + 210f(6x_k) - 252f(5x_k) \\ +210f(4x_k) - 120f(3x_k) + 45f(2x_k) - 3628810f(x_k))\|_k \leq \delta \quad (6)\end{aligned}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. Unifying (5) and (6),

$$\begin{aligned}\sup_{k\in\mathbb{N}} \|(10f(9x_1) - 55f(8x_1) + 120f(7x_1) - 165f(6x_1) + 252f(5x_1) \\ -330f(4x_1) + 120f(3x_1) - 1814235f(2x_1) + 3628810f(x_1), \\ \cdots, 10f(9x_k) - 55f(8x_k) + 120f(7x_k) - 165f(6x_k) + 252f(5x_k) \\ -330f(4x_k) + 120f(3x_k) - 1814235f(2x_k) + 3628810f(x_k))\|_k \leq \frac{3}{2}\delta \quad (7)\end{aligned}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. Letting $x_1, \cdots, x_k$ by $4x_1, \cdots, 4x_k$ and replacing $y_1, \cdots, y_k$ by $x_1, \cdots, x_k$ in (2), we arrive

$$\begin{aligned}\sup_{k\in\mathbb{N}} \|(f(9x_1) - 10f(8x_1) + 45f(7x_1) - 120f(6x_1) + 210f(5x_1) - 252f(4x_1) \\ +210f(3x_1) - 120f(2x_1) + 3628754f(x_1), \cdots, f(9x_k) - 10f(8x_k) \\ +45f(7x_k) - 120f(6x_k) + 210f(5x_k) - 252f(4x_k) + 210f(3x_k) \\ -120f(2x_k) - 3628754f(x_k))\|_k \leq \delta \quad (8)\end{aligned}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. Multiplying by 10 in (8), we arrive

$$\begin{aligned}\sup_{k\in\mathbb{N}} \|(10f(9x_1) - 100f(8x_1) + 450f(7x_1) - 1200f(6x_1) + 2100f(5x_1) \\ -2520f(4x_1) + 2100f(3x_1) - 1200f(2x_1) - 36287540f(x_1), \cdots, 10f(9x_k) \\ -100f(8x_k) + 450f(7x_k) - 1200f(6x_k) + 2100f(5x_k) \\ -2520f(4x_k) + 2100f(3x_k) - 1200f(2x_k) - 36287540f(x_k))\|_k \leq 10\delta \quad (9)\end{aligned}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. It folloys from (7) and (9), ye get

$$\begin{aligned}\sup_{k\in\mathbb{N}} \|(45f(8x_1) - 330f(7x_1) + 1035f(6x_1) - 1848f(5x_1) + 2190f(4x_1)\\ -1980f(3x_1) - 1813035f(2x_1) + 39916350f(x_1), \cdots, 45f(8x_k)\\ -330f(7x_k) + 1035f(6x_k) - 1848f(5x_k) + 2190f(4x_k) - 1980f(3x_k)\\ -1813035f(2x_k) + 39916350f(x_k))\|_k \leq \frac{23}{2}\delta \qquad (10)\end{aligned}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$.
Putting $x_1, \cdots, x_k$ by $3x_1, \cdots 3x_k$ and replacing $y_1, \cdots, y_k$ by $x_1, \cdots, x_k$ in (2), we get

$$\begin{aligned}\sup_{k\in\mathbb{N}} \|(f(8x_1) - 10f(7x_1) + 45f(6x_1) - 120f(5x_1) + 210f(4x_1) - 252f(3x_1)\\ +211f(2x_1) - 3628930f(x_1), \cdots, f(8x_k) - 10f(7x_k) + 45f(6x_k) - 120f(5x_k)\\ +210f(4x_k) - 252f(3x_k) + 211f(2x_k) - 3628930f(x_k))\|_k \leq \delta \qquad (11)\end{aligned}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. Multiplying by 45 in (11), we get

$$\begin{aligned}\sup_{k\in\mathbb{N}} \|(45f(8x_1) - 450f(7x_1) + 2025f(6x_1) - 5400f(5x_1) + 9450f(4x_1)\\ -11340f(3x_1) + 9495f(2x_1) - 163301850f(x_1), \cdots, 45f(8x_k)\\ -450f(7x_k) + 2025f(6x_k) - 5400f(5x_k) + 9450f(4x_k) - 11340f(3x_k)\\ +9495f(2x_k) - 163301850f(x_k))\|_k \leq 45\delta \qquad (12)\end{aligned}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. By (10) and (12), we obtain

$$\begin{aligned}\sup_{k\in\mathbb{N}} \|(120f(7x_1) - 990f(6x_1) + 355f(5x_1) - 7260f(4x_1) + 9360f(3x_1)\\ -1822530f(2x_1) + 203218200f(x_1), \cdots, 120f(7x_k) - 990f(6x_k) + 355f(5x_k)\\ -7260f(4x_k) + 9360f(3x_k) - 1822530f(2x_k) + 203218200f(x_k))\|_k \leq \frac{113}{2}\delta \qquad (13)\end{aligned}$$

for all $x_1, \cdots, x_k \in \mathcal{X}$.

Replacing $x_1, \cdots, x_k$ by $2x_1, \cdots, 2x_k$ and $y_1, \cdots, y_k$ by $x_1, \cdots, x_k$ in (2), we get

$$\sup_{k\in\mathbb{N}} \|(f(7x_1) - 10f(6x_1) + 45f(5x_1) - 120f(4x_1) + 211f(3x_1) - 262f(2x_1) \\ +3628545f(x_1), \cdots, f(7x_k) - 10f(6x_k) + 45f(5x_k) - 120f(4x_k) \\ +211f(3x_k) - 262f(2x_k) - 3628545f(x_k))\|_k \leq \delta \quad (14)$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. Multiplying by 120 on both sides in (14), we can get

$$\sup_{k\in\mathbb{N}} \|(120f(7x_1) - 1200f(6x_1) + 5400f(5x_1) - 14400f(4x_1) + 25320f(3x_1) \\ -31440f(2x_1) - 435425400f(x_1), \cdots, 120f(7x_k) - 1200f(6x_k) + 5400f(5x_k) \\ -14400f(4x_k) + 25320f(3x_k) - 31440f(2x_k) - 435425400f(x_k))\|_k \leq 120\delta \quad (15)$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. By (13) and (15), we get

$$\sup_{k\in\mathbb{N}} \|(210f(6x_1) - 1848f(5x_1) + 7140f(4x_1) - 15960f(3x_1) - 1791090f(2x_1) \\ +638643600f(x_1), \cdots, 210f(6x_k) - 1848f(5x_k) + 7140f(4x_k) \\ -15960f(3x_k) - 1791090f(2x_k) + 638643600f(x_k))\|_k \leq \frac{353}{2}\delta \quad (16)$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. Dividing on both sides by 2 in (16), we get

$$\sup_{k\in\mathbb{N}} \|(105f(6x_1) - 924f(5x_1) + 3570f(4x_1) - 7980f(3x_1) - 895545f(2x_1) \\ +319321800f(x_1), \cdots, 105f(6x_k) - 924f(5x_k) + 3570f(4x_k) \\ -7980f(3x_k) - 895545f(2x_k) + 319321800f(x_k))\|_k \leq \frac{353}{4}\delta \quad (17)$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. Replacing $y_1, \cdots y_k$ by $x_1, \cdots, x_k$ in (2), we get

$$\sup_{k\in\mathbb{N}} \|(f(6x_1) - 10f(5x_1) + 46f(4x_1) - 130f(3x_1) + 255f(2x_1) \\ -3629172f(x_1), \cdots, f(6x_k) - 10f(5x_k) + 46f(4x_k) - 130f(3x_k) \\ +255f(2x_k) - 3629172f(x_k))\|_k \leq \delta \quad (18)$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. Multiplying both sides 105 by (18), we can get

$$\sup_{k\in\mathbb{N}} \|(105f(6x_1) - 1050f(5x_1) + 4830f(4x_1) - 13650f(3x_1) + 26775f(2x_1)$$
$$-381063060f(x_1), \cdots, 105f(6x_k) - 1050f(5x_k) + 4830f(4x_k)$$
$$-13650f(3x_k) + 26775f(2x_k) - 3810630f(x_k))\|_k \leq 105\delta \quad (19)$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. From (17) and (19)

$$\sup_{k\in\mathbb{N}} \|(126f(5x_1) - 1260f(4x_1) + 5670f(3x_1) - 922320f(2x_1)$$
$$+700384860f(x_1) + 126f(5x_k) - 1260f(4x_k) + 5670f(3x_k)$$
$$-922320f(2x_k) + 700384860f(x_k))\|_k \leq \frac{773}{4}\delta \quad (20)$$

for all $x_1, \cdots, x_k \in \mathcal{X}$.

Replacing $x_1, \cdots, x_k = 0$ and $y_1, \cdots, y_k$ by $x_1, \cdots, x_k$ in (2), we obtain

$$\sup_{k\in\mathbb{N}} \|(2f(5x_1) - 20f(4x_1) + 90f(3x_1) - 240f(2x_1)+$$
$$-3628380f(x_1), \cdots, 2f(5x_k) - 20f(4x_k) + 90f(3x_k) - 240f(2x_k)$$
$$-3628380f(x_k))\|_k \leq \delta \quad (21)$$

for all $x_1, \cdots, x_k \in \mathcal{X}$.

Multiplying on bothsides by 65 in (21), we obtain that
$$\sup_{k\in\mathbb{N}} \|(126f(5x_1) - 1260f(4x_1) + 5670f(3x_1) - 15120f(2x_1) - 228587940f(x_1)$$
$$126f(5x_k) - 1260f(4x_k) + 5670f(3x_k) - 15120f(2x_k) - 228587940f(x_k))\|_k \leq 63\delta \quad (22)$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. From (20) and (22)

$$\sup_{k\in\mathbb{N}} \|(-907200f(2x_1) + 928972800f(x_1), \cdots,$$
$$-907200f(2x_k) + 928972800f(x_k))\|_k \leq \frac{1025}{4}\delta \quad (23)$$

for all $x_1, \cdots, x_k \in \mathcal{X}$. It follows from (23)that

$$\sup_{k\in\mathbb{N}} \|(f(2x_1) - 1024f(x_1), \cdots, f(2x_k) - 1024f(x_k))\|_k \leq \frac{205}{725760}\delta \quad (24)$$

$$\sup_{k\in\mathbb{N}}\left\|\left(f(x_1)-\frac{f(2x_1)}{2^{10}},\cdots,f(x_k)-\frac{f(2x_k)}{2^{10}}\right)\right\|_k \le \frac{41}{148635648}\delta \tag{25}$$

for all $x_1,\cdots,x_k \in \mathcal{X}$. Let $\Lambda = \{g:\mathcal{X}\to\mathcal{A}|g(0)=0\}$ and introduce the generalized metric d defined on λ by

$$d(o,p)=\inf\left\{\lambda\in[0,\infty]|\sup_{k\in\mathbb{N}}\|o(x_1)-p(x_1),\cdots,o(x_k)-p(x_k)\|_k\le\lambda \quad ll \quad x_1,\cdots,x_k\in\mathcal{X}\right\}$$

Then it is easy to show that Λ, d is a generalized complete metric space, See [11]. we define an operator $\mathcal{J}:\Lambda\to\Lambda$ by

$$\mathcal{J}o(x)=\frac{1}{2^{10}}0(2x) \qquad x\in\mathcal{X}.$$

we assert that $\mathcal{J}$ is a strictly contractive operator. Given $o,p\in\Lambda$, let $\lambda\in[0,\infty]$ be an arbitary constant yith $d(o,p)\le\lambda$. From the definition if follows that

$$\sup_{k\in\mathbb{N}}\|o(x_1)-p(x_1),\cdots,o(x_k)-p(x_k)\|_k\le\lambda \quad x_1,\cdots,x_k\in\mathcal{X}.$$

Therefore,

$$\begin{aligned}&\sup_{k\in\mathbb{N}}\|(\mathcal{J}o(x_1)-\mathcal{J}p(x_1),\cdots,\mathcal{J}o(x_k)-\mathcal{J}p(x_k))\|_k\\ &\quad\le\sup_{k\in\mathbb{N}}\left\|\left(\frac{1}{2^{10}}o(2x_1)-\frac{1}{2^{10}}p(2x_1),\cdots,\frac{1}{2^{10}}o(2x_k)-\frac{1}{2^{10}}p(2x_k)\right)\right\|_k\\ &\quad\le\frac{1}{2^{10}}\lambda\end{aligned}$$

$x_1,\cdots,x_k\in\mathcal{X}$. Hence,it holds that

$$d(\mathcal{J}o,\mathcal{J}p)\le\frac{1}{2^{10}}\lambda d(\mathcal{J}o,\mathcal{J}p)\le\frac{1}{2^{10}}d(o,p)$$

$llo,p\in\Lambda$.

This Means that $\mathcal{J}$ is strictly contractive operator on Λ with the Lipschitz constant $L=\frac{1}{2^{10}}$.

By (25), we have $d(\mathcal{J}f,f)\le\frac{41}{148635648}\delta$. According to Theorem 1.1, we deduce the existence of a fixed point of $\mathcal{J}$ that is the existence of mapping $\mathcal{D}:\mathcal{X}\to\mathcal{A}$ such that

$$\mathcal{D}(2x)=2^{10}\mathcal{D}(x) \qquad llx\in\mathcal{X}.$$

Moreover, we have $d\left(\mathcal{J}^n f, \mathcal{D}\right) \to 0$, which implies

$$\mathcal{D}(x) = \lim_{n\to\infty} \mathcal{J}^n f(x) = \lim_{n\to\infty} \frac{\xi(2^n x)}{2^{10n}}$$

for all $x \in \mathcal{X}$.

Also, $d(f, \mathcal{D}) \leq \dfrac{1}{1-L} d(\mathcal{J}f, f)$ implies the inequality

$$\begin{aligned} d(f, \mathcal{D}) &\leq \frac{1}{1 - \frac{1}{2^{10}}} d(\mathcal{J}f, f) \\ &\leq \frac{41}{148490496}\delta. \end{aligned}$$

Setting $x_1 =, \cdots, = x_k = 2^n x, y_1 =, \cdots, = y_k = 2^n y$ in (2) and divide both sides by 2^{10n}. Then, using property (a) of multi-norms, we obtain

$$\begin{aligned} \|\mathcal{G}\mathcal{D}(x,y)\| &= \lim_{n\to\infty} \frac{1}{2^{10n}} \|\mathcal{G}f\left(2^n x, 2^n y\right)\| \\ &\leq \lim_{n\to\infty} \frac{1}{2^{10n}} = 0 \end{aligned}$$

for all $x, y \in \mathcal{X}$. Hence $\mathcal{D}$ is Decic.

The uniqueness of $\mathcal{D}$ follows from the fact that $\mathcal{D}$ is the unique fixed point of $\mathcal{J}$ with the property that there exists $\ell \in (0, \infty)$ such that

$$\sup_{k\in\mathbb{N}} \|(f(x_1) - \mathcal{D}(x_1), \cdots, f(x_k) - \mathcal{D}(x_k))\|_k \leq \ell$$

for all $x_1, \cdots, x_k \in \mathcal{X}$.

This completes the proof of the Theorem. □

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