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Mean Cordial Labeling for Two Star Graphs

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Abstract. In this paper we prove that the two star $K_{1,g} \wedge K_{1,h}$ is mean cordial graph if and only if $|2g - h| \leq 4$ for $g \leq h$ and $g = 1, 2, 3, \cdots$.

Key words: Mean Cordial graph and star

Mathematics Subject classification 2010: 05C78.

1. Introduction

We begin with simple, finite, connected and undirected graph $G = (V(G), E(G))$ with order p and size q. The members of V(G) and E(G) are commonly termed as graph elements, while $|V(G)|$ and $|E(G)|$ denotes number of vertices and edges in graph G respectively.

In 1987, Cahit [1] have introduced cordial labeling. Let f be a function from the vertices of G to $\{0, 1\}$ and for each edge xy assigns the label $|f(x) - f(y)|$, call f a cordial labeling of G, if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 differ by atmost 1.

Raja Ponraj, Muthirulan Sivakumar and Murugesan Sundaram introduce a new notion called mean cordial labeling and they investigate the mean cordial labeling behavior of some standard graphs. The symbol $\lceil x \rceil$ stands for smallest integer greater than or equal to x.

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Definition 1.1. *Let f be a function from $V(G)$ to $\{0,1,2\}$ for each edge uv of G, assign the label* $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ *f is called a mean cordial labeling of G if* $|V_f(i) - V_f(j)| \leq 1$ *and* $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0,1,2\}$, *where* $V_f(x)$ *denotes the number of vertices and* $e_f(x)$ *denotes the number of edges labeled with* $x(x = 0,1,2)$ *respectively. A graph with a mean cordial labeling is called mean cordial graph.*

Definition 1.2. *A wedge is defined as an edge connecting two components of a graph, denoted as* $\wedge$, $\omega(G\wedge) < \omega(G)$.

Theorem 1.1. *The two star* $K_{1,g} \wedge K_{1,h}$ *is mean cordial graph if and only if* $|2g - h| \leq 4$ *for* $g \leq h$ *and* $g = 1,2,3,\cdots$.

proof Let G=$K_{1,g} \wedge K_{1,h}$.
$V(G)$ be the node set of G and $E(G)$ be the link set of G, then G is given by,
$V(G) = \{s,t\} \cup \{s_\theta : 1 \leq \theta \leq g\} \cup \{t_\theta : 1 \leq \theta \leq h\}$ and
$E(G) = \{ss_\theta : 1 \leq \theta \leq g\} \cup \{tt_\theta : 1 \leq \theta \leq h\} \cup \{s_\theta t_\theta$ for any $\theta\}$.
Then, G has $g + h + 2$ nodes and $g + h + 1$ links.
To prove that G is a mean cordial graph for all $g \geq 1$, $h \geq 1$
$f : V(G) \to \{0,1,2\}$ and $f^* : E(G) \to \{0,1,2\}$.
We shall consider the following cases.

Case(i): $h = 2g$

Consider the graph $G = K_{1,g} \wedge K_{1,h}$, where $g \leq h$.
The required node labeling of G is defined as follows: $f(s) = 0; f(t) = 1$
$f(s_\theta) = 0$ for $1 \leq \theta \leq g$
$f(t_{2\theta-1}) = 1$ for $1 \leq \theta \leq \frac{h}{2}$
$f(t_{2\theta}) = 2$ for $1 \leq \theta \leq \frac{h}{2}$
The required link labeling of G is defined as follows:
ss_θ is 0 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 1 for $1 \leq \theta \leq \frac{h}{2}$; $tt_{2\theta}$ is 2 for $1 \leq \theta \leq \frac{h}{2}$.
The wedge labeling of $s_\theta t_\theta$ is 1 for any θ.
Then, $v_f(0) = v_f(1) = g + 1$,$v_f(2) = g$ and $e_f(0) = e_f(2) = g$,$e_f(1) = g + 1$.
Hence, $|V_f(i) - V_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0,1,2\}$.
Hence, G is mean cordial graph if $h = 2g$.

Example: Let $g = 10$ then we get $h = 20$,i.e $K_{1,10} \wedge K_{1,20}$.

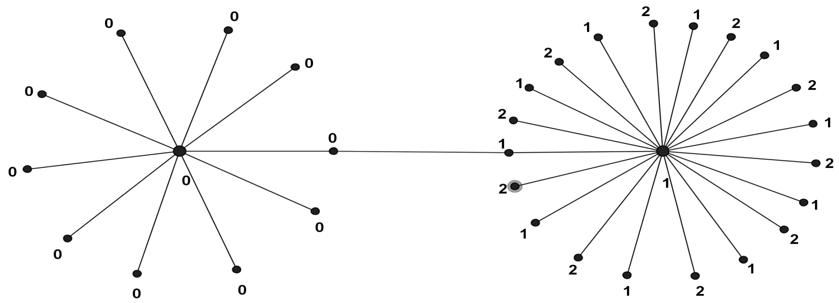

FIGURE 1. $K_{1,10} \wedge K_{1,20}$

Case(ii): $h = 2g + 1$

Consider the graph $G = K_{1,g} \wedge K_{1,h}$, where $g \leq h$.

The required node labeling of G is defined as follows:

$$f(s) = 0; f(t) = 1$$

$$\begin{array}{ll} f(s_\theta) = 0 & \text{for} \quad 1 \leq \theta \leq g \\ f(t_{2\theta-1}) = 1 & \text{for} \quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor \\ f(t_{2\theta}) = 2 & \text{for} \quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor \\ f(t_\theta) = 2 & \end{array}$$

The required link labeling of G is defined as follows:

ss_θ is 0 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 1 for $1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$; $tt_{2\theta}$ is 2 for $1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$.

The wedge labeling of $s_\theta t_\theta$ is 1 for any θ.

Then, $v_f(0) = v_f(1) = v_f(2) = g + 1$ and $e_f(0) = g$,$e_f(1) = e_f(2) = g + 1$.

Hence, $|V_f(i) - V_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$.

Hence, G is mean cordial graph if $h = 2g + 1$.

Example: Let $g = 10$ then we get $h = 21$,i.e $K_{1,10} \wedge K_{1,21}$.

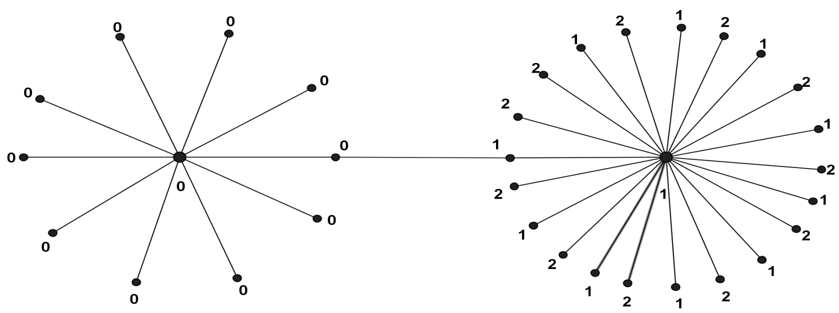

FIGURE 2. $K_{1,10} \wedge K_{1,21}$

Case(iii): $h = 2g + 2$

Consider the graph $G = K_{1,g} \wedge K_{1,h}$, where $g \leq h$.

The required node labeling of G is defined as follows:

$f(s) = 0; f(t) = 1$

$f(s_\theta) = 0 \quad$ for $\quad 1 \leq \theta \leq g$

$f(t_{theta}) = 0$

$f(t_{2\theta-1}) = 1 \quad$ for $\quad 1 \leq \theta \leq \frac{h}{2} - 1$

$f(t_{2\theta}) = 2 \quad$ for $\quad 1 \leq \theta \leq \frac{h}{2}$.

The required link labeling of G is defined as follows:

ss_θ is 0 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 1 for $1 \leq \theta \leq \frac{h}{2} - 1$; $tt_{2\theta}$ is 2 for $1 \leq \theta \leq \frac{h}{2}$.

The wedge labeling of $s_\theta t_\theta$ is 0 for any θ.

Then, $v_f(0) = g + 2$, $v_f(1) = v_f(2) = g + 1$ and $e_f(0) = e_f(1) = e_f(2) = g + 1$.

Hence, $|V_f(i) - V_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$.

Hence, G is mean cordial graph if $h = 2g + 2$.

Example: Let $g = 10$ then we get $h = 22$,i.e $K_{1,10} \wedge K_{1,22}$.

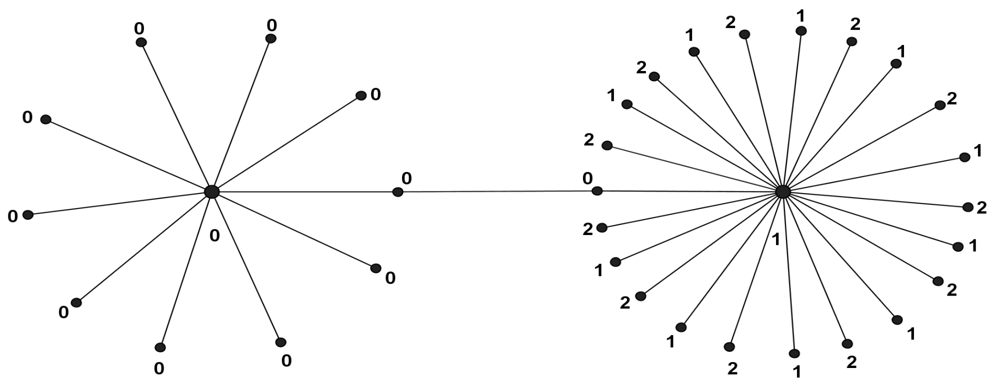

FIGURE 3. $K_{1,10} \wedge K_{1,22}$

Case(iv): $h = 2g + 3$

Consider the graph $G = K_{1,g} \wedge K_{1,h}$, where $g \leq h$.

The required node labeling of G is defined as follows: $f(s) = 0; f(t) = 1$

$f(s_\theta) = 0 \quad$ for $\quad 1 \leq \theta \leq g$

$f(t_\theta) = 0$

$f(t_{2\theta-1}) = 1 \quad$ for $\quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$

$f(t_{2\theta}) = 2 \quad$ for $\quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$

The required link labeling of G is defined as follows:

ss_θ is 0 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 1 for $1 \leq \theta \leq \lfloor \frac{h}{2} \rfloor$; $tt_{2\theta}$ is 2 for $1 \leq \theta \leq \lfloor \frac{h}{2} \rfloor$.

The wedge labeling of $s_\theta t_\theta$ is 0 for any θ

Then, $v_f(0) = v_f(1) = g+2$, $v_f(2) = g+1$ and $e_f(0) = e_f(2) = g+1$, $e_f(1) = g+2$.

Hence, $|V_f(i) - V_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$.

Hence, G is mean cordial graph if $h = 2g+3$.

Example: Let $g = 10$ then we get $h = 23$,i.e $K_{1,10} \wedge K_{1,23}$.

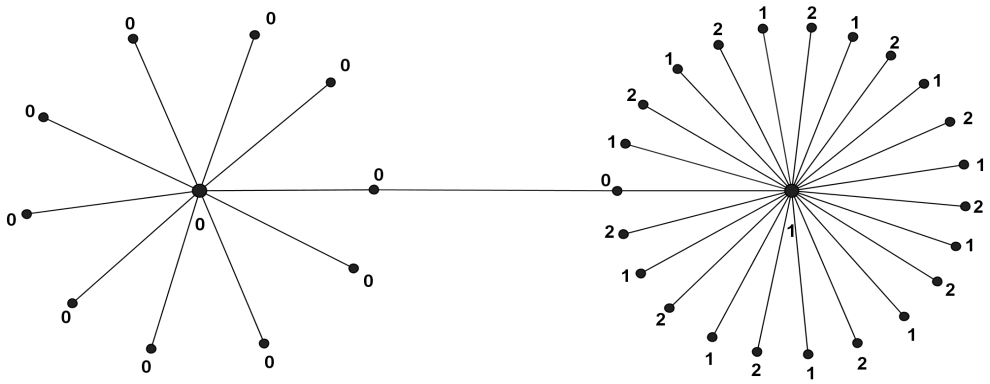

FIGURE 4. $K_{1,10} \wedge K_{1,23}$

Case(v): $h = 2g+4$

Consider the graph $G = K_{1,g} \wedge K_{1,h}$, where $g \leq h$.

The required node labeling of G is defined as follows:

$f(s) = 0; f(t) = 1$

$f(s_\theta) = 0 \quad \text{for} \quad 1 \leq \theta \leq g$

$f(t_{theta}) = 0$

$f(t_{2\theta-1}) = 1 \quad \text{for} \quad 1 \leq \theta \leq \frac{h}{2} - 1$

$f(t_{2\theta}) = 2 \quad \text{for} \quad 1 \leq \theta \leq \frac{h}{2}$

The required link labeling of G is defined as follows:

ss_θ is 0 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 1 for $1 \leq \theta \leq \frac{h}{2} - 1$; $tt_{2\theta}$ is 2 for $1 \leq \theta \leq \frac{h}{2}$.

The wedge labeling of $s_\theta t_\theta$ is 0 for any θ.

Then, $v_f(0) = v_f(1) = v_f(2) = g+2$ and $e_f(1) = e_f(2) = g+2$, $e_f(0) = g+1$.

Hence, $|V_f(i) - V_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$.

Hence, G is mean cordial graph if $h = 2g+4$.

Example: Let $g = 10$ then we get $h = 24$,i.e $K_{1,10} \wedge K_{1,24}$.

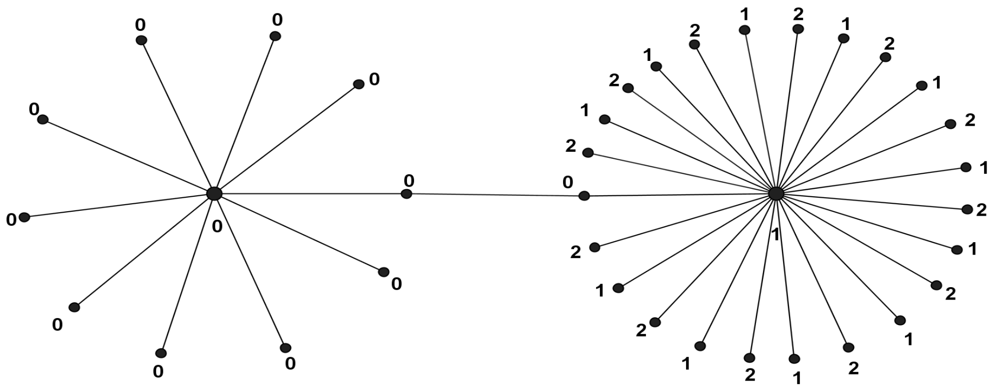

FIGURE 5. $K_{1,10} \wedge K_{1,24}$

Hence, G is mean cordial graph $|2g - h| \leq 4$ for $g \leq h$ and $g = 1, 2, 3, \cdots$ coveserly, we fix the 0 in s_θ where $1 \leq \theta \leq g$, some 0, 1 and 2 in t_θ, where $1 \leq \theta \leq h$, then only we get vertices less than or equal to 1.

Suppose, $h = 2g + 5$, consider the graph $G = K_{1,g} \wedge K_{1,h}$, where $g \leq h$

Suppose, if we fix, node labeling of G is defined as follows:

$f(s) = 0; f(t) = 0$

$$
\begin{aligned}
f(s_\theta) &= 0 \quad \text{for} \quad 1 \leq \theta \leq g \\
f(t_\theta) &= 0 \\
f(t_{2\theta-1}) &= 1 \quad \text{for} \quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor \\
f(t_{2\theta}) &= 2 \quad \text{for} \quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor
\end{aligned}
$$

The required link labeling of G is defined as follows:

ss_θ is 0 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 1 for $1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$; $tt_{2\theta}$ is 1 for $1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$.

The wedge labeling of $s_\theta t_\theta$ is 0.

Then, $v_f(0) = g + 3$,$v_f(1) = g + 2$, $v_f(2) = g + 2$ and $e_f(0) = g + 2$, $e_f(2) = 0$, $e_f(1) = 2g + 4$.

Hence, $|V_f(i) - V_f(j)| \leq 1$ but $|e_f(i) - e_f(j)| > 1$, $i, j \in \{0, 1, 2\}$, which is contradiction.

Suppose, if we fix, node labeling of G is defined as follows:

$f(s) = 1; f(t) = 1$

$f(s_\theta) = 0 \quad \text{for} \quad 1 \leq \theta \leq g$

$f(t_\theta) = 0$

$f(t_{2\theta-1}) = 1 \quad \text{for} \quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$

$f(t_{2\theta}) = 2 \quad \text{for} \quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$

The required link labeling of G is defined as follows:

ss_θ is 1 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 1 for $1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$; $tt_{2\theta}$ is 2 for $1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$.

The wedge labeling of $s_\theta t_\theta$ is 0.

Then, $v_f(0) = g+1$,$v_f(2) = g+2$, $v_f(1) = g+4$ and $e_f(0) = 1$,$e_f(1) = 2g+3$, $e_f(2) = g+2$.

Hence, $|V_f(i) - V_f(j)| > 1$ and $|e_f(i) - e_f(j)| > 1$, $i, j \in \{0, 1, 2\}$, which is contradiction.

Suppose, if we fix, node labeling of G is defined as follows:

$f(s) = 2; f(t) = 2$

$f(s_\theta) = 0 \quad \text{for} \quad 1 \leq \theta \leq g$

$f(t_\theta) = 0$

$f(t_{2\theta-1}) = 1 \quad \text{for} \quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$

$f(t_{2\theta}) = 2 \quad \text{for} \quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$

The required link labeling of G is defined as follows:

ss_θ is 1 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 2 for $1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$; $tt_{2\theta}$ is 2 for $1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$.

The wedge labeling of $s_\theta t_\theta$ is 0.

Then, $v_f(0) = g+1, v_f(1) = g+2$, $v_f(2) = g+4$ and $e_f(0) = 1$,$e_f(1) = g+1$, $e_f(2) = 2g+4$.

Hence, $|V_f(i) - V_f(j)| > 1$ and $|e_f(i) - e_f(j)| > 1$, $i, j \in \{0, 1, 2\}$, which is contradiction.

Suppose, if we fix, node labeling of G is defined as follows:

$f(s) = 0; f(t) = 1$

$f(s_\theta) = 0 \quad \text{for} \quad 1 \leq \theta \leq g$

$f(t_\theta) = 0$

$f(t_{2\theta-1}) = 1 \quad \text{for} \quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$

$f(t_{2\theta}) = 2 \quad \text{for} \quad 1 \leq \theta \leq \left\lfloor \frac{h}{2} \right\rfloor$

The required link labeling of G is defined as follows:

ss_θ is 1 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 1 for $1 \leq \theta \leq \lfloor \frac{h}{2} \rfloor$; $tt_{2\theta}$ is 2 for $1 \leq \theta \leq \lfloor \frac{h}{2} \rfloor$.

The wedge labeling of $s_\theta t_\theta$ is 0.

Then, $v_f(0) = v_f(2) = g+2$, $v_f(1) = g+3$ and $e_f(0) = g+1$,$e_f(1) = g+3$, $e_f(2) = g+2$.

Hence, $|V_f(i) - V_f(j)| \leq 1$ but $|e_f(i) - e_f(j)| > 1$, $i, j \in \{0,1,2\}$, which is contradiction.

Suppose, if we fix, node labeling of G is defined as follows:

$$f(s) = 0; f(t) = 2$$

$$\begin{aligned}
f(s_\theta) &= 0 && \text{for} \quad 1 \leq \theta \leq g \\
f(t_\theta) &= 0 \\
f(t_{2\theta-1}) &= 1 && \text{for} \quad 1 \leq \theta \leq \lfloor \tfrac{h}{2} \rfloor \\
f(t_{2\theta}) &= 2 && \text{for} \quad 1 \leq \theta \leq \lfloor \tfrac{h}{2} \rfloor
\end{aligned}$$

The required link labeling of G is defined as follows:

ss_θ is 0 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 2 for $1 \leq \theta \leq \lfloor \frac{h}{2} \rfloor$; $tt_{2\theta}$ is 2 for $1 \leq \theta \leq \lfloor \frac{h}{2} \rfloor$.

The wedge labeling of $s_\theta t_\theta$ is 0.

Then, $v_f(0) = v_f(1) = g+2$, $v_f(2) = g+3$ and $e_f(0) = g+1$,$e_f(1) = g+3$, $e_f(2) = 2g+4$.

Hence, $|V_f(i) - V_f(j)| \leq 1$ but $|e_f(i) - e_f(j)| > 1$, $i, j \in \{0,1,2\}$, which is contradiction.

Suppose, if we fix, node labeling of G is defined as follows:

$$f(s) = 1; f(t) = 2$$

$$\begin{aligned}
f(s_\theta) &= 0 && \text{for} \quad 1 \leq \theta \leq g \\
f(t_\theta) &= 0 \\
f(t_{2\theta-1}) &= 1 && \text{for} \quad 1 \leq \theta \leq \lfloor \tfrac{h}{2} \rfloor \\
f(t_{2\theta}) &= 2 && \text{for} \quad 1 \leq \theta \leq \lfloor \tfrac{h}{2} \rfloor
\end{aligned}$$

The required link labeling of G is defined as follows:

ss_θ is 1 for $1 \leq \theta \leq g$; $tt_{2\theta-1}$ is 2 for $1 \leq \theta \leq \lfloor \frac{h}{2} \rfloor$; $tt_{2\theta}$ is 2 for $1 \leq \theta \leq \lfloor \frac{h}{2} \rfloor$.

The wedge labeling of $s_\theta t_\theta$ is 0.

Then, $v_f(2) = v_f(1) = g + 3$, $v_f(0) = g + 1$ and $e_f(0) = 1$,$e_f(1) = g + 1$, $e_f(2) = 2g + 4$.

Hence, $|V_f(i) - V_f(j)| > 1$ and $|e_f(i) - e_f(j)| > 1$, $i, j \in \{0, 1, 2\}$, which is contradiction.

Hence, G is not mean cordial graph if $h = 2g + 5$.

Hence, the two star $K_{1,g} \wedge K_{1,h}$ is mean cordial graph if and only if $|2g - h| \leq 4$ for $g \leq h$ and $g = 1, 2, 3, \cdots ,$.

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