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Generalized Rational Inequalities in Complex Valued Metric Spaces

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Abstract. In this paper, the existence and uniqueness of common and coincidence fixed points for a pair of self mappings under generalized rational inequalities are established in complex valued metric spaces with supportive examples. The presented results improve and generalize the existing results in the literature.

Key words: Common fixed point, coincidence point, rational contraction, complex valued metric space.

AMS Subject classification: 05C15, 05C76.

1. Introduction and Preliminaries

Metric fixed point theory is widely recognized as one of the important traditional theories in nonlinear analysis that has a wide range of applications that have been originated in the PhD thesis of Banach [2] in 1922 where he proved the famous contraction mapping principle. Banach contraction principle assures the existence and uniqueness of a solution of an operator equation $Tx = x$, is the most widely used fixed point theorem in all of analysis. This principal is constructive in nature and is one of the most useful techniques in the concept of nonlinear equations. A great number of generalizations of the Banach contraction mapping principle were studied by many authors in metric spaces and some other spaces, see [3, 4, 6–13] and references given therein. These generalizations were made either by using the contractive condition or by imposing some additional conditions on an ambient space.

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Dass and Gupta [4] generalized Banach contraction principle under rational inequality. Later, Pachpatte [6] obtained the common fixed points for the mappings satisfying rational contractions. In 2011, Azam et al. introduced the concept of complex valued metric spaces and obtained some fixed point results for mappings satisfying a rational inequality. In [14], Saluja proved some fixed point theorems under rational contraction in the context of complex valued metric spaces. In spite of these results, in this paper, we prove the existence and uniqueness of common and coincidence fixed points for a pair of self mappings satisfying a generalized rational contractive condition in complex valued metric spaces. Examples are given to support the results obtained. The presented results improve and generalize the well known results in the literature.

We now recall the basic definitions and results that are required in the sequel.

Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\preceq$ on $\mathbb{C}$ as follows: $z_1 \preceq z_2$ if and only if $Re(z_1) \leq Re(z_2), Im(z_1) \leq Im(z_2)$. It follows that $z_1 \preceq z_2$ if one of the following conditions is satisfied:

(i) $Re(z_1) = Re(z_2), Im(z_1) < Im(z_2)$;

(ii) $Re(z_1) < Re(z_2), Im(z_1) = Im(z_2)$;

(iii) $Re(z_1) < Re(z_2), Im(z_1) < Im(z_2)$;

(iv) $Re(z_1) = Re(z_2), Im(z_1) = Im(z_2)$.

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \notin z_2$ and one of (i), (ii), or (iii) is satisfied and we will write $z_1 \prec z_2$ if only (iii) is satisfied. Note that

$$0 \preceq z_1 \precnsim z_2 \Rightarrow |z_1| < |z_2|,$$

$$z_1 \preceq z_2, z_2 \prec z_3 \Rightarrow z_1 \prec z_3.$$

In 2011, Azam et al. [1] introduced the following definition.

Definition 1.1. *Let X be a nonempty set. Suppose that the mapping $d : X \times X \to \mathbb{C}$ satisfies the following:*

(i) $0 \preceq d(x, y)$ *for all* $x, y \in X$ *with* $x \notin y$ *and* $d(x, y) = 0 \Longleftrightarrow x = y$;

(ii) $d(x, y) = d(y, x)$ *for all* $x, y \in X$*;*

(iii) $d(x, y) \preceq d(x, z) + d(z, y)$ *for all* $x, y, z \in X$.

Then d is called a complex valued metric on X and (X, d) is called a complex valued metric space.

Example 1.2. *Let $X = \mathbb{C}$, where $\mathbb{C}$ is the set of complex numbers. Define a mapping $d : X \times X \to \mathbb{C}$ by $d(z_1, z_2) = e^{it}|z_1 - z_2|$, where $z_1 = (x_1, y_1), z_2 = (x_2, y_2)$ and $t \in [0, \pi/2]$. Then (X, d) is a complex valued metric space.*

Definition 1.3. (i) *A point $x \in X$ is called an interior point of a subset $A \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that $B(x, r) = \{y \in X : d(x, y) \prec r\} \subseteq A$.*

(ii) *A point $x \in X$ is called a limit of A whenever for every $0 \prec r \in \mathbb{C}$ such that $B(x, r) \cap \Big(A - \{X\}\Big) \notin \emptyset$.*

(iii) *The set A is called open whenever each element of A is an interior point of A. A subset B is called closed whenever each limit point of B belongs to B.*

The family $F := \{B(x, r) : x \in X, 0 \prec r\}$ is a sub-basis for a Hausdorff topology τ on X.

Definition 1.4. *Let (X, d) be a complex valued metric space. Let $\{x_n\}$ be a sequence in X and $x \in X$. Then*

(i) *$\{x_n\}$ is called convergent, if for every $c \in \mathbb{C}$, with $0 \prec c$ there exists $n_0 \in \mathbb{C}$ such that for all $n > n_0, d(x_n, x) \prec c$. Also, $\{x_n\}$ converges to x (written as, $x_n \to x$ or $\lim\limits_{n\to\infty} x_n = x$) and x is the limit of $\{x_n\}$.*

(ii) *$\{x_n\}$ is called a Cauchy sequence in X, if for every $c \in \mathbb{C}$, with $0 \prec c$ there exists $n_0 \in \mathbb{N}$ such that for all $n > n_0, d(x_n, x_{n+m}) \prec c$.*

(iii) *If every Cauchy sequence converges in X, then X is called a complete complex valued metric space.*

Lemma 1.5. *[1] Let (X, d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $\lim\limits_{n\to\infty} |d(x_n, x)| = 0$.*

Lemma 1.6. *[1] Let (X, d) be a complex valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $\lim\limits_{m,n\to\infty} |d(x_n, x_{n+m})| = 0$.*

Definition 1.7. *Let X be a nonempty set and $S, T : X \to X$ be self mappings. Then*

(i) *an element $x \in X$ is said to be a fixed point of T if $x = Tx$.*

(ii) *If $Sx = Tx$, then $x \in X$ is called a coincidence point of T and f.*

(iii) *If $u = Sx = Tx$, then $u \in X$ is called a point of coincidence of T and f.*

(iv) *If $x = Sx = Tx$ then x is called a common fixed point of S and T.*

(v) *The mappings S and T are said to be commuting if $S(Tx) = T(Sx)$ for all $x \in X$.*

(vi) *The mappings S and T are said to be weakly compatible if they commute at their coincidence points.*

Proposition 1.8. *Let f and g be weakly compatible self mappings on a non empty set X. If f and g have a unique point of coincidence $v = fu = gu$, then v is the unique common fixed point of f and g.*

2. Main Results

The following is one of the main results of this section.

Theorem 2.1. *Let (X, d) be a complex valued metric space and $S, T : X \to X$ be two self mappings such that $T(X) \subseteq S(X)$ and $S(X)$ is a complete subspace of X. Suppose that*

$$\begin{aligned} d(Tx, Ty) \ \preceq \ & \mu \max\left\{ d(Sx, Sy), \frac{d(Sx, Tx)d(Sy, Ty)}{1 + d(Sx, Sy)}, \right. \\ & \left. \frac{d(Sx, Tx)d(Sy, Ty)}{1 + d(Tx, Ty)} \right\} \end{aligned} \tag{1}$$

for all $x, y \in X$, $x \neq y$, where $\mu < 1$. Then T and S have a unique point of coincidence in X. In addition, if T and S are weakly compatible then T and f have a unique common fixed point in X.

Proof. Let $x_0 \in X$ be an arbitrary point. Since $T(X) \subset S(X)$, there exists $x_1 \in X$ such that $Tx_0 = Sx_1$. Proceeding in this fashion, for $x_n \in X$, we get $x_{n+1} \in X$ such that $Tx_n = Sx_{n+1}$, $n = 0, 1, 2, \ldots$. Then, using (3), we obtain

$$\begin{aligned}
d(Sx_{n+1}, Sx_n) &\preceq d(Tx_n, Tx_{n-1}) \\
&\preceq \mu \max\left\{ d(Sx_n, Sx_{n-1}), \frac{d(Sx_n, Tx_n)d(Sx_{n-1}, Tx_{n-1})}{1 + d(Sx_n, Sx_{n-1})}, \right. \\
&\left. \frac{d(Sx_n, Tx_n)d(Sx_{n-1}, Tx_{n-1})}{1 + d(Tx_n, Tx_{n-1})} \right\} \\
&= \mu \max\left\{ d(Sx_n, Sx_{n-1}), \frac{d(Sx_n, Sx_{n+1})d(Sx_{n-1}, Sx_n)}{1 + d(Sx_n, Sx_{n-1})}, \right. \\
&\left. \frac{d(Sx_n, Sx_{n+1})d(Sx_{n-1}, Sx_n)}{1 + d(Sx_{n+1}, Sx_n)} \right\}.
\end{aligned}$$

Case 1: If the maximum is $d(Sx_n, Sx_{n-1})$, then

$$d(Sx_{n+1}, Sx_n) \preceq \mu d(Sx_n, Sx_{n-1}),$$

where $\mu < 1$. Continuing in this way, we obtain

$$d(Sx_{n+1}, Sx_n) \preceq \mu d(Sx_n, Sx_{n-1}) \preceq \mu^2 d(Sx_{n-1}, Sx_{n-2}) \preceq \ldots \preceq \mu^n d(Sx_0, Sx_1).$$

For $m > n$, we have

$$\begin{aligned}
d(Sx_m, Sx_n) &\preceq d(Sx_m, Sx_{m-1}) + d(Sx_{m-1}, Sx_{m-2}) + \cdots + d(Sx_{n+1}, Sx_n), \\
&\preceq (\mu^{m-1} + \mu^{m-2} + \cdots + \mu^n) d(Sx_1, Sx_0), \\
&\preceq \frac{\mu^n}{1-\mu} d(Sx_1, Sx_0).
\end{aligned}$$

Finally, we have

$$|d(Sx_m, Sx_n)| \leq \frac{\mu^n}{1-\mu} |d(Sx_1, Sx_0)| \to 0 \text{ as } m, n \to \infty.$$

Case 2: If the maximum is $\dfrac{d(Sx_n, Sx_{n+1})d(Sx_{n-1}, Sx_n)}{1 + d(Sx_n, Sx_{n-1})}$, then it is easy to note that it will bring a contraction to our assumption.

Case 3: If the maximum is $\dfrac{d(Sx_n, Sx_{n+1})d(Sx_{n-1}, Sx_n)}{1 + d(Sx_{n+1}, Sx_n)}$, then it can be seen that

$$\begin{aligned}
d(Sx_{n+1}, Sx_n) &\preceq \mu d(Sx_n, Sx_{n-1}) - 1, \\
&\preceq \mu^2 d(Sx_{n-1}, Sx_{n-2}) - \mu - 1, \\
&\preceq \ldots \preceq \mu^n d(Sx_1, Sx_0) - (1 + \mu + \mu^2 + \ldots + \mu^{n-1}).
\end{aligned}$$

Since $\mu < 1$, for $m > n$ we have $|d(Sx_m, Sx_n)| \to 0$ as $m, n \to \infty$.

Thus, from all three cases, it can be shown that $\{Sx_n\}$ is a Cauchy sequence in $S(X)$. Because of the completeness of $S(X), Sx_n \to v$ and there exists $u \in X$ such that $Su = v$. Further, we have

$$\begin{aligned}
d(Sx_n, Tu) &= d(Tx_{n-1}, Tu) \\
&\preceq \mu \max\left\{ d(Sx_{n-1}, Su), \frac{d(Sx_{n-1}, Tx_{n-1})d(Su, Tu)}{1 + d(Sx_{n-1}, Su)}, \right. \\
&\left. \frac{d(Sx_{n-1}, Tx_{n-1})d(Su, Tu)}{1 + d(Tx_{n-1}, Tu)} \right\} \\
&= \mu \max\left\{ d(Sx_{n-1}, Su), \frac{d(Sx_{n-1}, Sx_n)d(Su, Tu)}{1 + d(Sx_{n-1}, Su)}, \right. \\
&\left. \frac{d(Sx_{n-1}, Sx_n)d(Su, Tu)}{1 + d(Sx_n, Tu)} \right\}
\end{aligned}$$

Now, letting $n \to \infty$, we obtain

$$|d(v, Tu)| \leq 0,$$

which implies that $Tu = v = Su$, that is, v is a point of coincidence of T and S. Now we claim that v is a unique point of coincidence of T and S. Suppose there exists another point of coincidence v' of T and S, that is, $Tu' = v' = Su'$. Then, from (3), we have

$$\begin{aligned}
d(Su, Su') &= d(Tu, Tu') \\
&\preceq \mu \max\left\{ d(Su, Su'), \frac{d(Su, Tu)d(Su', Tu')}{1 + d(Su, Su')}, \right. \\
&\left. \frac{d(Su, Tu)d(Su', Tu')}{1 + d(Tu, Tu')} \right\}
\end{aligned}$$

which gives

$$|d(Su, Su')| \leq a|d(Su, Su')|.$$

Thus, $Su = Su'$ which shows that v is a unique point of coincidence of T and S. Now, by Proposition 1.8, T and f have a unique common fixed point in X. $\square$

Remarks 2.2.

(i) *Theorem 2.1 generalizes and improves Theorem* 4 *of Azam et al. [1], Theorem* 3.1 *of Saluja [14].*

(ii) *Moreover, Theorem 2.1 guarantees the existence and uniqueness of coincidence point of S and T.*

We obtain the following result by taking $S = I_X$ (the identity mapping) in Theorem 2.1.

Corollary 2.3. *Let (X, d) be a complete complex valued metric space and $T : X \to X$ be a self mapping such that*

$$\begin{aligned} d(Tx, Ty) \;\; &\preceq \;\; \mu \max\left\{ d(x, y), \frac{d(x, Tx)d(y, Ty)}{1 + d(x, y)}, \right. \\ &\left. \frac{d(x, Tx)d(y, Ty)}{1 + d(Tx, Ty)} \right\} \end{aligned} \tag{2}$$

for all $x, y \in X,\ x \neq y$, where $\mu < 1$. Then T has a unique fixed point in X.

Theorem 2.4. *Let (X, d) be a complex valued metric space and $S, T : X \to X$ be two self mappings such that $T(X) \subseteq S(X)$ and $S(X)$ is a complete subspace of X. Suppose that*

$$\begin{aligned} d(Tx, Ty) \;\; &\preceq \;\; ad(Sx, Sy) + b\frac{d(Sx, Tx)d(Sy, Ty)}{1 + d(Sx, Sy)} \\ &+ c\frac{d(Sx, Tx)d(Sy, Ty)}{1 + d(Tx, Ty)} \end{aligned} \tag{3}$$

for all $x, y \in X,\ x \neq y$, where a, b, c are non negative constants such that $a+b+c < 1$. Then T and S have a unique point of coincidence in X. In addition, if T and S are weakly compatible then T and S have a unique common fixed point in X.

Corollary 2.5. *Let (X, d) be a complete complex valued metric space and $T : X \to X$ be a self mapping such that*

$$\begin{aligned} d(Tx, Ty) \;\; &\preceq \;\; ad(x, y) + b\frac{d(x, Tx)d(y, Ty)}{1 + d(x, y)} \\ &+ c\frac{d(x, Tx)d(y, Ty)}{1 + d(Tx, Ty)} \end{aligned} \tag{4}$$

for all $x, y \in X,\ x \neq y$, where a, b, c are non negative constants such that $a+b+c < 1$. Then T has a unique fixed point in X.

Corollary 2.6. *Let (X, d) be a complex valued metric space and $S, T : X \to X$ be two self mappings such that $T(X) \subseteq S(X)$ and $S(X)$ is a complete subspace of X. Suppose that*

$$d(Tx, Ty) \preceq ad(Sx, Sy)$$

for all $x, y \in X,\ x \neq y$, where a is a non negative constant such that $a < 1$. Then T and S have a unique point of coincidence in X. Moreover, if T and S are weakly compatible then T and S have a unique common fixed point in X.

Putting $S = I_X$ (the identity mapping) in the previous corollary, we get the following Banach Contraction Principle [2] in the framework of complex metric spaces.

Corollary 2.7. *Let (X, d) be a complete complex valued metric space and $T : X \to X$ be a self mappings such that*

$$d(Tx, Ty) \preceq ad(x, y)$$

for all $x, y \in X,\ x \neq y$, where $0 < a < 1$. Then T has a unique fixed point in X.

Theorem 2.8. *Let (X, d) be a complex valued metric space and $S, T : X \to X$ be two self mappings such that $T^n(X) \subseteq S^n(X)$ and $S^n(X)$ is a complete subspace of X. Suppose that the following holds for some fixed $n \in \mathbb{N}$*

$$\begin{aligned} d(T^n x, T^n y) \quad &\preceq \quad \mu \max\left\{ d(S^n x, S^n y), \frac{d(S^n x, T^n x) d(S^n y, T^n y)}{1 + d(S^n x, S^n y)}, \right. \\ &\left. \frac{d(S^n x, T^n x) d(S^n y, T^n y)}{1 + d(T^n x, T^n y)} \right\} \end{aligned} \tag{5}$$

for all $x, y \in X,\ x \neq y$, where $\mu < 1$. Then T and S have a unique point of coincidence in X. In addition, if T and S are weakly compatible then T and f have a unique common fixed point in X.

Proof. It follows that $T^n u = S^n u = u$ from Theorem 2.1, for fixed $n \in \mathbb{N}$. Then

$$\begin{aligned} d(Su, Tu) &= d(ST^n u, TT^n u) = d(T^n Su, T^n Tu) \\ &\precsim \mu \max\left\{ d(S^n Su, S^n Tu), \frac{d(S^n Su, T^n Su) d(S^n Tu, T^n Tu)}{1 + d(S^n Su, S^n Tu)}, \right. \\ &\left. \frac{d(S^n Su, T^n Su) d(S^n Tu, T^n Tu)}{1 + d(T^n Su, T^n Tu)} \right\} \\ &= \mu \max\left\{ d(SS^n u, TS^n u), \frac{d(SS^n u, ST^n u) d(TS^n u, TT^n u)}{1 + d(SS^n u, TS^n u)}, \right. \\ &\left. \frac{d(SS^n u, ST^n u) d(TS^n u, TT^n u)}{1 + d(ST^n u, TT^n u)} \right\} \end{aligned}$$

which yields that

$$\begin{aligned} |d(Su, Tu)| = \mu \max\left\{ |d(Su, Tu)|, \frac{|d(Su, Su)||d(Tu, Tu)|}{1 + |d(Su, Tu)|}, \right. \\ \left. \frac{|d(Su, Su)||d(Tu, Tu)|}{1 + |d(Su, Tu)|} \right\} \end{aligned}$$

Finally, we get

$$|d(Su, Tu)| \leq 0,$$

which implies that $Tu = v = Su$, that is, v is a point of coincidence of T and S. Further, using the similar arguments as of Theorem 2.1, it can be viewed that u is the unique common fixed point of S and T. □

If the second and third terms inside the max in (5) are 0 in Theorem 2.8, we obtain the following corollary which can be viewed as an extension of the results obtained by Bryant [3] and Saluja [14] in complex valued metric space.

Corollary 2.9. *Let (X, d) be a complex valued metric space and $S, T : X \to X$ be two self mappings such that $T^n(X) \subseteq S^n(X)$ for $n \in \mathbb{N}$ and $S^n(X)$ is a complete subspace of X. Suppose that*

$$d(T^n x, T^n y) \ \precsim \ ad(S^n x, S^n y) \tag{6}$$

for all $x, y \in X$, $x \neq y$, where a is a non negative constant such that $a < 1$. Then T and S have a unique point of coincidence in X.

Corollary 2.10. *Let (X, d) be a complete complex valued metric space and $T : X \to X$ be a self mappings such that for fixed $n \in \mathbb{N}$*

$$d(T^n x, T^n y) \preceq a d(x, y)$$

for all $x, y \in X$, $x \neq y$, where $0 < a < 1$. Then T has a unique fixed point in X.

Example 2.11. *Let $X = \mathbb{C}$ and define a mapping $d : \mathbb{C} \times \mathbb{C} \to \mathbb{C}$ by $d(z_1, z_2) = |x_1 - x_2| + i|y_1 - y_2|$ where $z_1 = x_1 + iy_1, z_2 = x_2 + iy_2$. Then $(\mathbb{C}, d)$ is a complex valued metric space. Now define $T : \mathbb{C} \to \mathbb{C}$ as follows*

$$T(x+iy) = \begin{cases} 0, & if\ x, y \in Q \\ i, & if\ x, y \in Q^c \\ 1, & if\ x \in Q^c, y \in Q \\ 1+i, & if\ x \in Q, y \in Q^c \end{cases}$$

Note that, for $x = \frac{1}{\sqrt{5}}$ and $y = 1$ we obtain the following

$$d\left(T(\frac{1}{\sqrt{5}}), T(1)\right) = d(1, 0) \preceq \mu d\left(\frac{1}{\sqrt{5}}, 1\right) = \frac{\mu}{\sqrt{5}}$$

which implies that $1 \leq \frac{\mu}{\sqrt{5}} \Rightarrow \sqrt{5} \leq \mu$ which is a contradiction to the fact that $\mu < 1$. However, one can note that $T^2(z) = 0$ for all z. So

$$0 = d(T^2(z_1), T^2(z_2)) \preceq \mu d(z_1, z_2)$$

which shows that T^2 satisfies all the conditions Corollary 2.10. Hence $z = 0$ is a unique fixed point of T.

Example 2.12. *Let $X = \{0, 1, 2\}$ and define a partial order $\preceq$ as $x \preceq y$ iff $x \geq y$. Let the complex valued metric d be given as follows:*

$$d(x, y) = |x - y|(1 + i) \quad \forall x, y \in X$$

and $S, T : X \to X$ as

$$T(0) = 0, T(1) = 1, T(2) = 1 \text{ and } S(0) = 0, S(1) = 0, S(2) = 2.$$

Case 1: If $x = 0, y = 1$, *then it can be easily seen that* (3) *and other conditions of Theorem 2.1 are satisfied.*

Case 2: If $x = 0, y = 2$, *then*

$$d(T(0), T(2)) = (1 + i) \leq \mu 2(1 + i),$$

which implies that (3) *and other conditions of Theorem 2.1 are satisfied for* $1/2 \leq \mu < 1$.

Case 3: Taking $x = 1, y = 2$, *implies that* (3) *and other conditions of Theorem 2.1 are satisfied. Hence,* $0 \in X$ *is a unique common fixed point of* S *and* T.

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