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Hausdorff Property of Transformation Graphs

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Abstract. A Hausdorff graph G is a simple graph in which any two vertices u and v of G satisfy atleast one of the following conditions: (i) both u and v are isolated vertices (ii) either u or v is an isolated vertex (iii) there exists two non-adjacent edges e_1 and e_2 of G such that e_1 is incident with u and e_2 is incident with v. In this paper, we investigate Hausdorff property on transformation graphs.

Key words: Hausdorff Graph, Transformation Graph.

Mathematics Subject classification 2010: 05C76, 05C99.

1. Introduction

In [5], eight types of transformation graph were introduced and their basic properties were studied. Several authors have worked on these eight types of transformation graph separately. In [2], B. Wu, L. Zhang, Z. Zhang obtained a necessary and sufficient condition for G^{-++} to be hamiltonian.

Let $G = (V(G),\ E(G))$ be a simple undirected graph and x, y, z be three variables taking values $+$ or $-$. The transformation graph G^{xyz} is the graph having $V(G) \cup E(G)$ as a vertex set, and two vertices α and β of G^{xyz} are adjacent if and only if one of the following conditions holds: (i) for $\alpha,\ \beta \in V(G)$, α and β are adjacent in G if $x = +$; α and β are not adjacent in G if $x = -$ (ii) for $\alpha,\ \beta \in E(G)$, α and β are adjacent in G if $y = +$; α and β are not adjacent in G if $y = -$ (iii) for $\alpha \in V(G)$, $\beta \in E(G)$, α and β are incident in G if $z = +$; α and β are not incident in G if $z = -$.

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Seena and Raji introduced Hausdorff properties in [3] and they discussed Hausdorff property of some derived graphs. A Hausdorff graph G is a simple graph in which any two vertices u and v of G satisfy atleast one of the following conditions: (i) both u and v are isolated vertices (ii) either u or v is an isolated vertex (iii) there exists two non-adjacent edges e_1 and e_2 of G such that e_1 is incident with u and e_2 is incident with v. Terms not defined are used in the sense of [1]. We use the following theorems for proving main results.

Theorem 1.1. [5] *For a graph G, $G^{+yz} \cong G$ if and only if G is an empty graph.*

Theorem 1.2. [2] *For a graph G, G^{-++} is Hamiltonian if and only if $|V(G)| \geq 3$.*

Theorem 1.3. [3] *Any Hamiltonian graph with more than 3 vertices is Hausdorff.*

In this paper, we obtain results on $G^{+++}, G^{---}, G^{-++}$ and G^{+--}.

2. Main Result

In this section, we obtain necessary and sufficient condition for G^{+++} to be Hausdorff, sufficient condition for G^{---} to be non-Hausdorff and sufficient condition for G^{-++} and G^{+--} to be Hausdorff.

Theorem 2.1. *The transformation graaph G^{+++} of a graph G is Hausdorff if and only if G has no copy of K_2 as a component.*

Proof. Assume that G has no copy of K_2 as a component. Let $u_1,\ u_2$ be two distinct vertices of G^{+++}.

case 1: $u_1,\ u_2 \in V(G)$.

Subcase (i) u_1 and u_2 are isolated in G.

Then u_1 and u_2 are isolated in G^{+++}.

Subcase (ii) u_1 or u_2 is isolated in G.

Then u_1 or u_2 is isolated in G^{+++}.

Subcase (iii) u_1 and u_2 are adjacent vertices of G.

Let $e_1 = u_1u_2$. By hypothesis, there exists a vertex u_3 such that u_3 is adjacent to either u_1 or u_2. Suppose that u_3 is adjacent to u_1 in G. Then clearly u_2e_1 and u_1u_3 are two non-adjacent edges of G^{+++}. Suppose that u_3 is adjacent to u_2 in G. Then clearly u_1e_1 and u_2u_3 are two non-adjacent edges of G^{+++}.

Subcase (iv) u_1 and u_2 are non adjacent vertices of G.

Since u_1 and u_2 are not isolated, there exists two distinct edges e_1 and e_2 such that e_1 is incident with u_1 and e_2 is incident with u_2.Then u_1e_1 and u_2e_2 are two non-adjacent edges of G^{+++}.

Case 2: $u_1,\ u_2 \in E(G)$.

Since $u_1 \neq u_2$, there exists two distinct vertices u_3 and u_4 such that u_1 is incident with u_3 and u_2 is incident with u_4. Then u_1u_3 and u_2u_4 are two non-adjacent edges of G^{+++}.

Case 3: $u_1 \in V(G)$ and u_2(say e_1) $\in E(G)$.

Subcase (i) e_1 is incident with u_1 in G.

Let $e_1 = u_1u_3$ be an edge of G. By hypothesis, there exists an edge e_2 different from e_1 such that e_1 and e_2 are adjacent in G. Then e_1e_2 and u_1u_3 are two non-adjacent edges of G^{+++}.

Subcase (ii) e_1 is not incident with u_1 in G.

Suppose u_1 is isolated in G. Then u_1 is isolated in G^{+++}. Suppose u_1 is not isolated in G. Then there exists an edge e_2 incident with u_1 in G. Let u_3 be one endpoint of e_1 in G. Then clearly u_1e_2 and e_1u_3 are two non-adjacent edges of G^{+++}. Thus G^{+++} is Hausdorff.

Conversely, Assume that G^{+++} is Hausdorff. Suppose K_2 is one of the component of G. Let u_1, u_2 be two distinct vertices of G^{+++}. Suppose $u_1, u_2 \in V(K_2)$. Let $e_1 = u_1u_2$ be an edge of K_2. Then $V(G^{+++}) \supseteq \{e_1, u_1, u_2\}$. For these three vertices of G^{+++}, hausdorff property is not true. Therefore G has no copy of K_2 as a component. □

Theorem 2.2. *The transformation graph G^{---} of a graph G is not Hausdorff if $G \cong K_{1,r} \cup K_1$ or $G \cong K_{1,r} + e$ where e is an edge and $r \geq 1$.*

Proof. Suppose $G \cong K_{1,r} \cup K_1$. Then G consists of an isolated vertex u_1 and a vertex u_2 such that u_2 is adjacent to every other vertices of G other than u_1.Hence by the definition of the transformation graph G^{---}, u_2 is adjacent to u_1 in G^{---} and no other vertices of G^{---} is adjacent to u_2. So $deg\ u_2 = 1$ in G^{---}. Therefore G^{---} is not Hausdorff.
Suppose $G \cong K_{1,r} + e$. Then G consists of a vertex u such that u is adjacent to every other vertices of G and e is an edge of G not incident with u. Hence by the definition of the transformation graph G^{---}, u is adjacent to e in G^{---} and no other vertices of G^{---} is adjacent to u. So $deg\ u = 1$ in G^{---}. Therefore G^{---} is not Hausdorff. □

Remark 1. *The converse of the above theorem need not be true. For a graph K_3^c, G^{---} is not Hausdorff.*

Theorem 2.3. *Let G be any graph of order $n \geq 4$. Then G^{-++} is Hausdorff.*

Proof. By Theorem 1.2, G^{-++} is Hamiltonian. Hence by Theorem 1.3, it is Hausdorff. □

Theorem 2.4. *If G is an empty graph then G^{+yz} is Hausdorff.*

Proof. By Theorem 1.1, $G^{+yz}$is an empty graph. Hence it is Hausdorff. □

Theorem 2.5. *Let G be any graph of order $n \geq 4$. If G has no copy of K_2 as a component then G^{+--} is Hausdorff.*

Proof. Let u_1 and u_2 be two distinct vertices of G^{+--}. Suppose G is a empty graph, then G^{+--} is a empty graph. Therefore G^{+--} is Hausdorff. Suppose G is not a empty graph.

Case 1: $u_1,\ u_2 \in V(G)$.

Subcase (i) u_1 and u_2 are isolated in G.

Since G is not a empty graph, there exists at least one edge e_1. By hypothesis, there exists an edge e_2 such that e_2 is adjacent to e_1. Then u_1e_1 and u_2e_2 are two non-adjacent edges of G^{+--}.

Subcase (ii) u_1 or u_2 is isolated in G.

Suppose u_1 is isolated in G. Since u_2 is not islated in G, there exists a vertex u_3 such that u_3 is adjacent to u_2. Let $e_1 = u_2u_3$. Then u_1e_1 and $u_2u_3$are two non-adjacent edges of G^{+--}.

Subcase (iii) u_1 and u_2 are adjacent vertices of G.

Let $e_1 = u_1u_2$. By hypothesis, there exists another edge e_2 of G which is incident with either u_1 or u_2. Let us take $e_2 = u_1u_3$. Then $u_1u_3$and u_2e_2 are two non-adjacent edges of G^{+--}.

Subcase (iv) u_1 and u_2 are non-adjacent vertices of G.

Since $u_1 \neq u_2$, there exists two distinct edges e_1 and e_2 such that e_1 is incident with u_1 and $e_2$is incident with u_2. Then u_1e_2 and u_2e_1 are two non-adjacent edges of G^{+--}.

Case 2: $u_1,\ u_2 \in E(G)$.

Since $u_1 \neq u_2$, there exists two distinct vertices u_3 and u_4 of G such that u_1 is incident with u_3 and u_2 is incident with u_4. Then u_1u_4 and u_2u_3 are two non-adjacent edges of G^{+--}.

Case 3: $u_1 \in V(G), u_2\ (= e_1) \in E(G)$.

Subcase (i) e_1 is incident with u_1 in G.

Let $e_1 = u_1u_3$. By hypothesis, there exists an edge e_2 different from e_1 such that e_2 is incident with u_1 or u_3. Let us suppose that e_2 is adjacent to u_1. Let us take $e_2 = u_1u_4$. Then u_1u_3 and e_1u_4 are two non-adjacent edges of G^{+--}.

Subcase (ii) e_1 is not incident with u_1 in G.

Suppose u_1 is isolated in G. Let $e_1 = u_3u_4$. By hypothesis, there exists an edge e_2 which is incident with either u_3 or u_4. Let us take $e_2 = u_4u_5$. Then u_1e_2 and e_1u_3 are two non-adjacent edges of G^{+--}. Suppose u_1 is not isolated in G. Then u_1 is adjacent to a vertex u_3 in G. Let us take $e_2 = u_1u_3$. Let $u_4$be an endpoint of e_1 in G. Suppose e_1 is adjacent to e_2 in G. Then e_1 is incident with u_3. Since $n \geq 4$, there exists a vertex u_5 not incident with e_1 other than u_1. Then clearly u_1u_3 and e_1u_5 are two non-adjacent edges of G^{+--}. Suppose e_1 is not adjacent to e_2 in G. Then u_1u_3 and e_1e_2 are two non-adjacent edges of G^{+--}.

Thus by all the above cases G^{+--} is Hausdorff. □

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